Dispersion of speckle suppression efficiency for binary DOE structures: spectral domain and coherent matrix approaches

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Abstract: We present the first general theoretical description of speckle suppression efficiency based on an active diffractive optical element (DOE). The approach is based on spectral analysis of diffracted beams and a coherent matrix. Analytical formulae are obtained for the dispersion of speckle suppression efficiency using different DOE structures and different DOE activation methods. We show that a one-sided 2D DOE structure has smaller speckle suppression range than a two-sided 1D DOE structure. Both DOE structures have sufficient speckle suppression range to suppress low-order speckles in the entire visible range, but only the two-sided 1D DOE can suppress higher-order speckles. We also show that a linear shift 2D DOE in a laser projector with a large numerical aperture has higher effective speckle suppression efficiency than the method using switching or step-wise shift DOE structures. The generalized theoretical models elucidate the mechanism and practical realization of speckle suppression.

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References and links

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1. Introduction

Lasers diodes are compact and energy efficient light sources that emit high quality beams [1]. These properties are most important for portable technical devices, such as nano- or pico-projectors and some medical test systems. However, image creation by a laser is modulated by speckles, which strongly decrease image quality [2]. Speckle is the main reason for limited application of laser illumination in many technical devices [3-4], especially portable projectors [5-6]. Therefore, various methods have been developed for speckle suppression [6–11], among which the method based on diffractive optical element (DOE) is one of the most important and simplest methods for speckle reduction. The method uses active DOE to decrease the spatial coherence of a laser beam by means of changing the intensity and the phase of diffraction orders (referred to as angle diversity).

The method based on DOE was firstly introduced in 1998 for speckle reduction in a laser projection system [11]. Thereafter, many works have been done to improve the DOE structures and optimize DOE activities for more effective speckle suppression and miniaturize the laser projection systems [12–20]. Two kinds of DOE structures, the one-sided 2D DOE (OSDOE) and the two-sided 1D DOE (TSDOE), have been examined for the speckle contrast and speckle suppression range [12–15]. In addition, binary phase codes with pseudorandom sequences [13], Barker codes [16,18-19] and the Hadamard matrix [10,17] have been developed for DOEs used in laser projection systems for speckle suppression. Furthermore, various activation methods, such as mechanical movements including linear shift, rotation and step-wise shift, as well as methods based on switching DOE structures, have been investigated for decorrelating laser beams [12-13,17–19,21-22]. It was shown that DOE based on binary pseudorandom sequences can be realized as 1D and 2D DOE structures and both structures can use linear shift to achieve the best speckle suppression [18-19]. DOE methods based on Hadamard matrix use a sequence of different DOE structures, each one of which is obtained from one Hadamard matrix column, to generate uncorrelated laser beams and has been demonstrated a very efficient speckle suppression method [10,17,21-22]. However, mechanical movements are not an optimal solution for methods based on Hadamard matrix, because the DOE in this method has a complex structure and it would require complex DOE movement to generate decorrelated beam sequences. Hence, electrical switching of DOE...
phase profiles was investigated for DOE methods based on Hadamard matrix. It can eliminate the need for complicated structures and operations. Although the well-known optically active media (e.g., liquid crystals [14] have so far too slow of a response for applications of speckle suppression, more suitable novel media (e.g., digital micro-mirror devices [23-24]) will be adopted for fast binary modulation in speckle suppression by DOE.

As we know, the laser projector uses red, green and blue lasers to create a color picture. Using only one DOE for all three lasers is the best technological solution for speckle suppression. It has been shown that an OSDOE based on pseudorandom sequences has narrower speckle suppression range than the visible range [12-13,15]. It has also been shown that a TSDOE has speckle suppression range wider than the visible range, and therefore, it can be the optimal solution of speckle suppression problem [12–15]. It is important to know whether this property is true for any OSDOE and any TSDOE, for example for a DOE based on Hadamard matrix. Therefore, a general theoretical model for calculating the dispersion of speckle suppression efficiency is vital for various DOE structures.

In spite of the relatively large number of publications devoted to the topic of speckle suppression, there is no any universal mathematical model that can analyze dispersion of speckle suppression of different DOE structures by far. A simple analytical formula for dispersion of speckle suppression efficiency was obtained only for linear-shifting TSDOE based on pseudorandom sequences [18-19]. The simulations of speckle suppression for OSDOE and TSDOE based on pseudorandom sequences [13,15] required a large number of calculations to obtain dispersion curves. Besides, for methods based on the Hadamard matrix, there is only a simple algorithm for calculation of speckle suppression efficiency at the central wavelength (the wavelength at which DOE gives exactly a \( \pi \) phase shift of the laser beam wavefront) [10].

Considering that the structure of a DOE and its activation pattern are the two essential issues for the performance of speckle suppression, we thoroughly study the dependency between speckle contrast (SC) with DOE structure and activation from the perspective of universal application. Firstly, we investigate the relationship between diffraction order and the decorrelated laser beam for different DOE movements, including linear shift, step-wise shift and switching DOE structures. Then, we study the dispersion of speckle suppression efficiency for different DOE structures with different DOE activation patterns. Finally, we develop a general approach with a derived formula system for analyzing the dispersion of speckle suppression efficiency based on the method using DOE shift placed inside of the optical system of projector. This first description of a generalized theoretical models for speckle suppression based on active DOEs will be helpful for understanding speckle suppression mechanisms and the practical processes of speckle suppression using the methods based on an active DOE.

**2. Optical scheme of speckle suppression by an active DOE**

The optical scheme with an active DOE structure is shown in Fig. 1. The collimated laser beam passes through the DOE. The objective lens gathers diffracted light and creates an image of the DOE on the screen. The active DOE change the phase profile of the diffracted beam and decrease the beam’s spatial coherence. The input numerical aperture of the objective lens should be large enough to gather almost all light diffracted from the DOE. In addition, the input aperture of the human eye (or camera) should be small enough to avoid resolving an image of one DOE period (primitive cell) on screen.
We assume that the DOE based on pseudorandom binary sequences is a periodic structure that has a period $T_0$ and elementary cell of width $T$ ($T_0/T = N$). All DOE cell transverse dimensions are multiples of the elementary cell width $T$. We also assume that one period of the 2D DOE structure (primitive cell) has square shape with the side length of $T_0 = N \cdot T$ (the results obtained below can be easily extended for a DOE with a rectangular primitive cell).

For speckle suppression based on Hadamard matrix, it is assumed that, at any given moment, we have the periodic structure based on the same primitive cell (made from the same Hadamard matrix column). The subsequent DOE-switching gives a sequence of different DOE structures (switching between different Hadamard matrix columns) that produce a sequence of decorrelated laser beams at the central wavelength.

A binary OSDOE structure is a two-level (land and groove) periodic DOE structure on one side of a transparent plate (see Fig. 2(a)). It is also assumed that at the central wavelength, the structure height gives exactly a $\pi$ phase shift between the light that passes through groove and land.

In the case of a pseudorandom based DOE, the structure of the DOE is defined from sequence of ‘1’ and ‘0’ elements of pseudorandom sequence. The wavefront phase shift on 0 and $\pi$ corresponds to elements ‘0’ and ‘1’ of the pseudorandom sequences, respectively. The phase of the elementary cell of a 2D DOE is defined from a square matrix. Every element of the matrix column is the result of XOR operation of an $M$-sequence with sequential element (equal to a number of columns of the same sequences). Figure 2(a) shows one period of this DOE structure based on a binary $M$-sequence with code length $N = 15$. The grey and white areas in Fig. 2(a) mean $\pi$ and 0 phase shift, correspondingly. The TSDOE structure is two 1D DOEs placed on opposite sides of a transparent plate, as shown in Fig. 2(b)-Fig. 2(d). Every 1D DOE has areas with 0 and $\pi$ phase shift in correspondence to an $M$-sequence code. Figure 2(b)-Fig. 2(d) show the TSDOE based on $M$-sequences with code length $N = 15$. 
The Hadamard matrix OSDOE is a sequence of periodic DOE having a square primitive cell. The primitive cell is created from the column of Hadamard matrix. Hadamard matrix is a binary matrix which consists of 1 and −1 digits [25]. The columns and rows of Hadamard matrix are orthogonal to each other. They have many other unique properties and therefore found application in many information technologies [25]. The wavefront phase shift on 0 and π corresponds to element ‘1’ and ‘-1’ of the Hadamard matrix, respectively. It was shown that DOE based on the Sylvester-Hadamard matrix has TSDOE realization [17]. Figure 3 shows the sequence of DOE structure based on the Sylvester-Hadamard matrix with an order of \( N_0 = N \cdot N = 2^{2m} = 16, N = 2^m = 4 \). It should be noted that since we used square DOE structures, the order of the Sylvester-Hadamard matrix must be \( N_0 = 2^{3m} \), where \( m \) is any positive integer number. It is evident that the upper row and left column can be used for the decomposition of 2D DOE sequence in sequence of the TSDOE structure.

Fig. 3. Sequence of sixteen DOE structures based on Sylvester-Hadamard matrix [10].

Figures 4(a) and 4(b) show liquid crystal (optically active medium) realizations of OSDOE and TSDOE structures [19], respectively, for the case of pseudorandom binary sequences based DOE. The same realizations can be easily constructed for the case of a Sylvester-Hadamard matrix based DOE. The amplitude of the applied voltage should be chosen to provide a π wavefront shift at the central wavelength.

Fig. 4. Liquid crystal realization of OSDOE (a) and TSDOE (b) structure based on pseudorandom binary sequences with \( N = 7 \).
3. Relation between the diffraction order of DOE and decorrelated laser beams

3.1 Spatial frequency approach for a method using DOE-shift for speckle suppression

The speckle contrast \( SC \) is used for evaluation of the amplitude of speckle noise, which is determined as:

\[
SC = \frac{\Delta I}{\overline{I}},
\]

where \( \Delta I \) and \( \overline{I} \) are the standard deviation and the average intensity in the image with homogenously illuminated beams, respectively. The coefficient of speckle suppression \( k \) is usually used to measure the efficiency of speckle suppression:

\[
k = \frac{SC_0}{SC},
\]

where \( SC_0 \) and \( SC \) are the initial speckle contrast and the speckle contrast after application of speckle suppression method, respectively.

The \( SC \) is determined by the number of beams that produce decorrelated laser speckle, and can be calculated by the formula [2]:

\[
SC = \frac{\sqrt{\sum_{n=1}^{N} I_n^2}}{\sum_{n=1}^{N} I_n},
\]

where \( I_n \) is intensity of \( n \)-th decorrelated beam. The speckle suppression efficiency can be calculated easily if the relation between the intensity of decorrelated laser beams and intensity of diffraction orders is determined.

It is assumed that the optical scheme is optimized for speckle suppression. Therefore, the angles between neighboring diffraction orders are large enough to create a decorrelated speckle pattern \( \lambda/(GT_0) > d/(2Z_3) \) [2] (they must be decorrelated in some way), where \( G = Z_2/Z_1 \) is optical magnification of projector, \( d \) is eye pupil, \( Z_3 \) is distance from eye to screen (see Fig. 1). Since it is assumed that the DOE has period \( T_0 \) and elementary cell of length \( T = T_0/N \), the diffraction beam has a divergence angle of \( \varphi = \lambda/T \) with approximately \( 2N \) diffraction orders \( \varphi_i = \varphi_i/T_0 = \varphi_i/NT, I = -N, \ldots, 0, \ldots, N \) inside the divergent angle and with approximately \( 2N \cdot 2N \) diffraction orders inside the solid angle of \( 2\varphi_i \cdot 2\varphi_i \). It should be noted that there are also higher diffraction orders with low intensity outside the divergent angle, which are involved in the speckle suppression mechanism.

From Eq. (1), we have \( SC \approx 1/\sqrt{N} \) if the intensity of all decorrelated beams is equal. It is well known that \( SC \) has a broad minimum level of \( SC < 1/\sqrt{N} \), when all decorrelated beams have an equal intensity. However, if the intensity of partial diffraction orders dominates others, \( SC \) would be significantly larger than the minimum value. Usually, the incoherent beams have different intensity, and therefore the number of decorrelated laser beams not always correctly shows the \( SC \). The efficient number of decorrelated laser beams (\( N_{ed}^{1D} \)) is defined as:

\[
SC = \frac{\sqrt{\sum_{n=1}^{N} I_n^2}}{\sum_{n=1}^{N} I_n} = 1/\sqrt{N_{ed}^{1D}}.
\]

\( N_{ed}^{1D} \) can be used for calculating the speckle suppression efficiency of the method. The diffracted beams by a 1D DOE diverge only in one plane and cannot use the whole aperture for speckle suppression. Therefore, the analysis of speckle suppression below will focus on methods using 2D DOE, though some obtained results will be also valid for 1D DOE. In the case of a 2D DOE with a square primitive cell, the diffracted light propagates in all directions. Therefore, we define the efficient number \( N_{ed} \) of decorrelated laser beams and rewrite Eq. (4) as:
\[ SC = \sqrt{\sum_{i=0}^{N_{0}-1} I_{i}^{2} / \sum_{i=0}^{N_{0}-1} I_{i}} = 1 / \sqrt{N_{0} \cdot N_{0}} = 1 / N_{0}. \]  

(5)

where \( N_{0} \) determines the spatial spectral domain with diffraction orders of \( N_{0} = N_{d} \cdot N_{f} \). It is clear that the effective number of diffraction orders coincides with the speckle suppression coefficient \( k \) (see Eq. (2)) in the case of a 2D DOE. However, to highlight the origin of the method, we will use the term of effective number of diffraction orders below.

### 3.2 Decorrelation of laser beams by an active DOE

The relation between diffraction orders and the number of decorrelated laser beams that illuminate the screen depends on the method of DOE activation. Any method of DOE shifts that fulfills large speckle suppression is important and interesting. In the case of using a 1D DOE, it is assumed that the DOE shifts by one DOE period during intensity integration time.

For the method using 2D DOE linear shift for speckle suppression, it will be assumed that the DOE shifts horizontally and has a small inclination angle \( \tan(\alpha) = 1/mN \) of the primitive cell relative to the direction of the DOE shift. It is also assumed that DOE shifts a distance \( L = NmT_{0} \) during the intensity integration time of the photo sensor (exposure time), as well as at a distance of one DOE-period \( T_{0} \) in a direction orthogonal to the fast DOE shift due to inclination angle.

In the case of step-wise moving, we assume that DOE shifts instantly from one position to another by one elementary DOE cell. The DOE shifts \( N \cdot N \) times and one elementary cell takes all possible positions in the DOE primitive cell during intensity integration time.

It was shown in \([10, 17]\) that \( N \cdot N \) switching of Hadamard matrix results in a speckle contrast of \( 1/\sqrt{N \cdot N} \). Since only uncorrelated beams can produce this minimum level of \( SC \), every DOE structure switch should produce an uncorrelated laser beam. Therefore, it will be assumed below that in the method based on Hadamard matrix type DOE, every DOE structure that switches at the central wavelength leads to radiation of uncorrelated laser beam. The consistent switching should result in a sequence radiation of all possible decorrelated laser beams during intensity integration time (human eye resolution time).

#### 3.2.1 Decorrelation of laser beams by linear DOE shift

It is assumed that during intensity integration time, DOE shifts at a distance \( L = mNT_{0} \) with inclination angle of \( \tan(\alpha) = 1/mN \), and \( m \gg 1 \). In this case, the field correlation coefficient between the fields of different diffraction orders at the back plane of DOE can be calculated as follows:
Hence, fields of all diffraction orders are decorrelated and the average intensity therefore can be calculated by:

\[
\left \langle I(x) \right \rangle = \sum_{n=-N}^{N} \sum_{m=-N}^{N} |a_{nm}|^2.
\]

where \(x\) and \(y\) are transverse coordinates at the back plane of the DOE, \(s\) is DOE displacement during the a relevant time interval, \(v\) is speed of DOE movement, the indices \(n, m, n_1, m_1\) are diffraction orders, and \(a_{nm}\) is the amplitude of the diffraction order. From Eq. (7), it follows that the intensity of light at the back plane of the DOE is the sum of intensities of all diffraction orders. Hence, a DOE shift decorrelates the light of all diffraction orders. It can be shown that the average light intensity on a screen also will be equal to the sum of the intensities of diffraction orders that passed through the projector objective. Since all diffraction orders are incident on the screen under decorrelation angles, every diffraction order will create a decorrelated speckle pattern.

### 3.2.2 Decorrelation of laser beams by step-wise DOE movement

For step-wise movement, the average intensity at the rear side of DOE is the sum of the light intensity in every fixed position of DOE, which can be written as follows:

\[
\left \langle I(x) \right \rangle = \left \langle E^*(x,y)E(x,y) \right \rangle = \frac{1}{N^2} \sum_{l=-N}^{N} \sum_{j=-N}^{N} E^*(x+lT, y+jT)E(x+lT, y+jT).
\]

where \(E(x, y)\) is the field amplitude, \(l\) and \(j\) are the numbers of DOE shifts by one elementary cell length along the \(x\) and \(y\) axis, respectively. It can be easily shown that:

\[
\left \langle \exp \left \{ i \left \langle \frac{2\pi n}{N} (x+vt) \right \rangle \right \} \exp \left \{ -i \left \langle \frac{2\pi m}{N} (y+v \tan(\alpha)) \right \rangle \right \} \right \rangle = \frac{1}{2N-1} \sum_{l=0}^{2N-1} \exp \left \{ i \left \langle \frac{2\pi n}{N} (x' + lT) \right \rangle \right \} \exp \left \{ -i \left \langle \frac{2\pi m}{N} (y' + lT \tan(\alpha)) \right \rangle \right \} = \frac{1}{2N-1} \exp \left \{ i \left \langle \frac{2\pi n}{N} (x' - x) \right \rangle \right \} \left( 1 - \exp \left \{ 2\pi (n-m) \right \} \right) \left( 1 - \exp \left \{ 2\pi (n-m) \right \} \right) \left( 1 - \exp \left \{ 2\pi (n-m) \right \} \right) \left( 1 - \exp \left \{ 2\pi (n-m) \right \} \right)
\]

since...
\[
\frac{1}{2N-1} \left( \frac{1}{1 - \exp\left(i2\pi \frac{n-m}{N} \right)} \right) = \begin{cases} 
\text{if } n-m = N \ell \text{ is any integer} \\
0; \text{ all other case}
\end{cases}
\]  

(10)

The average intensity of laser beam at the back plane of 1D DOE can be written as follows:

\[
\langle I(x) \rangle = \sum_{n=0}^{N-1} |f_n(x)|^2,
\]

(11)

\[
f_n(x) = \sum_{l=0}^{l_0} a_{n+l,N} \exp\left(i2\pi (n+LN) \cdot x / T_o \right).
\]

where \( l_0 \) is number of highest diffraction order of DOE. Similarly, it is easy to obtain the average intensity at the back plane of 2D DOE:

\[
\langle I(x,y) \rangle = \sum_{n=-1}^{N-2} \sum_{l=0}^{l_0} \left( \sum_{m=0}^{N-1} \sum_{j=0}^{N-1} a_{n+m,N-1} \exp\left(i2\pi (n+LN) \cdot x / T_o \right) \exp\left(i2\pi (m+LN) \cdot y / T_o \right) \right).
\]

(12)

\[
f_{mn}(x,y) = \sum_{n=-1}^{N-2} \sum_{l=0}^{l_0} a_{m+n,N-1} \exp\left(i2\pi (N+n) \cdot x / T_o \right) \exp\left(i2\pi (m+n) \cdot y / T_o \right).
\]

(13)

It should be noted that, for intensity calculations on the screen, the summation should be over diffraction orders that passed through the projector objective and illuminate the screen (not over all diffraction orders). Since every component of the sum (right hand side of Eq. (7)) creates an uncorrelated speckle pattern (difference between diffraction orders is larger than decorrelation angle), we have \( N \) and \( N \cdot N \) decorrelated speckle pattern for a 1D DOE and 2D DOE, respectively.

Based on the above analysis, the method based on linear shift with small inclination angle decorrelates all diffraction orders that illuminate the screen. The method based on sequence of step-wise DOE movement by one DOE elementary cell decorrelates at most \( N \cdot N = T_o / T \) laser beams, each of which is a linear combination of different diffraction orders.

3.2.3 Decorrelated laser beams by switching DOE structures

The method based on Hadamard matrix uses DOE structure switching to create \( N_o = N \cdot N \) DOE structures, each of which in equal time interval is used for screen illumination and has the same intensity. There are already many different realizations reported [10, 17]. For the analysis of 2D DOE (OSDOE), all realizations are equivalent and therefore should produce the same SC. We hereafter analyze DOE based on Sylvester-Hadamard matrix, since it has two-sided 1D (TSDOE) realization [17]. One period of this 2D DOE structure for the case \( N = 4 \) is shown in Fig. 3. The first matrix is a simple parallel plane plate that has only zero diffraction order. All other DOEs have the same area of groove and land and therefore the diffracted light has zero intensity of 00 diffraction order at the central frequency. It was proved that the method gives the minimum possible value of \( SC = 1/N \). Therefore, every DOE structure at the central wavelength produces a decorrelated speckle pattern in eye (emits decorrelated beams). The field at the eye retina can be written as follows:
\[ E(x, y, t) = \sum_{m=0}^{N} F_{nm}(x, y) \Pi\left(t - (nN + m)\Delta t\right)/\Delta t. \]  

(14)

and \( F_{nm}(x, y) \) is the decorrelated field amplitude that creates decorrelated speckle pattern, \( \Pi \) is rectangular function.

4. Dispersion of speckle suppression efficiency

4.1 Dispersion of speckle suppression efficiency for the case of DOE shift

It was shown above that speckle suppression efficiency by moving a DOE is determined by the number and intensity of diffraction orders that illuminate the screen (by the efficient number of diffraction orders). The DOE shift results in the decorrelation of diffraction orders. In all methods mentioned above for speckle suppression, we found the expansion of the illuminated field in a set of decorrelated laser beams at the central wavelength. In every expansion, a decorrelated beam coincides with zero diffraction order and all fields of other beams are superposition of higher diffraction order fields. One needs to find how decorrelated intensities alter with a change of laser wavelength to calculate the dispersion of \( SC \). The change of the laser wavelength forces light energy to flow from all higher diffraction orders to zero order or vice versa. The amplitude of zero diffraction order for 2D DOE structure changes as:

\[ A_{00} = a_{00} \cos(\phi). \]  

(15)

and the amplitude of any higher \( ij \)-th diffraction order mode changes as:

\[ A_{ij} = a_{ij} \sin(\phi). \]  

(16)

where \( A_{00} \) and \( A_{ij} \) are amplitudes of zero and higher diffraction orders, respectively, \( \phi = kh(n_{oi} - 1)/2, k = 2\pi/\lambda \) is wave number, \( h \) is DOE structure height, \( \lambda \) is laser wavelength, \( n_{oi} \) is refractive index of DOE structure, \( a_{00} \) and \( a_{ij} \) are amplitudes of zero and higher diffraction orders at central frequency (\( \cos(\phi) = 0, \sin(\phi) = 1 \)), respectively.

The power of all higher diffraction orders fast decreases and approaches to zero when the laser wavelength shifts from the central wavelength. Therefore, any binary DOE structure at a wavelength that differs markedly from the central wavelength should have a large intensity of zero diffraction order that dominates all other diffraction orders, which cause the decrease of speckle suppression efficiency. Since high efficiency speckle suppression requires homogeneous power distribution between all diffraction orders, so the intensity of zero diffraction order should be close to zero at the central wavelength.

4.1.1 OSDOE shift

According to the ratiocination above, any 2D DOE structure with efficient speckle suppression should be designed in such a way that the amplitude of zero diffraction order is close to zero at the central wavelength. Hereafter, it will be assumed that DOE has zero intensity for zero diffraction order. Since all higher diffraction orders have the same dispersion, we can simplify the diffraction pattern and replace them by \( N_{ef}^2 - 1 \), where \( N_{ef} \) should be calculated at the central wavelength and all diffraction orders have the same intensity. For this case, the amplitude of zero and higher diffraction orders can be written as follows:

\[ I_{00} = I \cos^2(\phi). \]  

(17)

\[ I_{ij} = I \sin^2(\phi)/(N_{ef}^2 - 1). \]  

(18)
where $I_i$ is the initial intensity of laser beam (before diffraction). By substituting Eqs. (17) and (18) into Eq. (3), we obtained a simple formula for SC of 2D DOE structure:

$$SC = \sqrt{\left(\frac{I_0}{N_0^2-1}\right)^2 + \left(\frac{I_i}{N_i^2-1}\right)^2}.$$  \hspace{1cm} (19)

### 4.1.2 TSDOE shift

In the case of a TSDOE, the intensity dispersion of diffraction orders is different. After the laser beam is diffracted by the first 1D DOE, the intensity of diffraction orders can be written as follows:

$$I_i = I_i \cos^2(\varphi).$$  \hspace{1cm} (20)

and after the diffraction by second 1D DOE, we have:

$$I_{ii} = I_{ii} \cos^2(\varphi).$$  \hspace{1cm} (22)

By substituting Eqs. (22)-(24) into Eq. (3) we obtain SC for TSDOE as:

$$SC = \sqrt{\cos^2(\varphi) + 2 \cos^2(\varphi) \sin^2(\varphi) / (N_{ef}^2 - 1)^2}.$$  \hspace{1cm} (25)

It should be noted that $N_{ef}$ is different for linear and step-wise DOE shift, as shown above.

### 4.1.3 Binary pseudorandom sequences with a small code length

In this section, the case of a DOE based on binary pseudorandom sequences with small code length ($N \leq 5$) is discussed. It is easy to see that a DOE based on binary pseudorandom sequences with small code length $N \leq 5$ has a relatively large intensity of the zeroth diffraction order at the central wavelength. Therefore, accurate calculation of speckle suppression for this case requires that we take into the consideration the intensity of the zeroth diffraction order at the central wavelength. The accurate formula for amplitude of zero diffraction order for OSDOE with small $N$ can be obtained as follows:

$$A_0 = n_+ + n_- \exp(i2\varphi).$$  \hspace{1cm} (26)

where $n_+$ and $n_-$ are the number of land and groove elementary cells in one DOE primitive cell, respectively. $n_+ + n_- = N^2$, $N$ is code length. The intensity of zero diffraction order and all other effective diffraction orders in this case can be calculated by using the formulæ:

$$I_m = \left\{ n_+ - n_- \right\}^2 + 4n_+ n_- \cos^2(\varphi) / N^4,$$

$$I_m = 4n_+ n_- \sin^2(\varphi) / N^4.$$

and for the SC, we have:

$$SC = \sqrt{\left((n_+ - n_-)^2 + 4n_+ n_- \cos^2(\varphi) / (N^2 - 1)^2\right)} / N^4.$$  \hspace{1cm} (27)

For the case of TSDOE based on pseudorandom binary sequences with small code length ($N \leq 5$), the intensity of zero and higher diffraction orders after being diffracted by the first 1D DOE can be calculated as:
\[ I_\varphi = I \left[ \frac{(N-1) + \exp(i2\varphi)}{N} \right] \left[ \frac{(N-1) + \exp(-i2\varphi)}{N} \right] = I \left( N-2 \right)^2 + 4(N-1)\cos^2(\varphi) \] \hspace{1cm} (29)

\[ I_\varphi = 4(N-1)\sin^2(\varphi) / \left[ N^2 (N_\phi - 1) \right]. \] \hspace{1cm} (30)

and after the diffraction of the second 1D DOE, we obtain:

\[ I_{oo} = I \left[ (N-2)^2 + 4(N-1)\cos^2(\varphi) \right] / N^4. \] \hspace{1cm} (31)

\[ I_{on} = 4I_\varphi \sin^2(\varphi)(N-1) \left[ (N-2)^2 + 4(N-1)\cos^2(\varphi) \right] / \left[ N^4 (N_\phi - 1) \right]. \] \hspace{1cm} (32)

\[ I_{on} = 16I_\varphi \sin^4(\varphi) / \left[ N^4 (N_\phi - 1)^2 \right]. \] \hspace{1cm} (33)

In Eqs. (29)-(33), it was taken into account that for the small code length of \( N \leq 5 \), only one elementary cell of one period of a 1D Barker-code DOE is located at the groove level. In this case, SC can be written as follows:

\[ SC = \frac{1}{N^4} \left[ \left( (N-2)^2 + 4(N-1)\cos^2(\varphi) \right)^2 + 256(N-1)^4\sin^4(\varphi) \left( (N_\phi - 1)^2 + \frac{32\sin^4(\varphi)(N-1) \left[ (N-2)^2 + 4(N-1)\cos^2(\varphi) \right]^2}{(N_\phi - 1)} \right) \right]. \] \hspace{1cm} (34)

4.2 Dispersion of speckle suppression efficiency for switching between different DOE structures

4.2.1 OSDOE based on Hadamard matrix

In the method of speckle suppression using DOE based on Hadamard matrix, the screen at the central wavelength is illuminated by a sequence of decorrelated laser beams (at separate times) that are formed by a sequence of different DOE structures. Since all the beams have the same intensity, \( N_0 = N_\phi^2 = N^2 \), when the laser wavelength shifts from the central wavelength, the beams within the whole illuminated screen except zero diffraction order change and the field of any beam can be written as follows:

\[ F_{on}(x,y) = \exp(i\varphi_{on})F_{on}(x,y)\sin(\varphi) + F_{00} \cos(\varphi). \] \hspace{1cm} (35)

where \( F_{on}(x,y) \) and \( F_{00} \) have the same intensity. It is evident that, in this case, the fields of different DOE structures are not fully decorrelated, and we cannot directly apply Eq. (3) for speckle contrast calculation. We have to use the coherence matrix technique \([2, 26]\) to derive the formula for SC. The coherence matrix is determined as follows [2]:

\[ M = \left( \left( F^* \right)^T : F^{*T} \right). \] \hspace{1cm} (36)

where \( M \) is coherence matrix, \( F^* \) is vector column consisting of components \( F_{00}^*, F_{01}^*, \ldots, F_{N-1,N-1}^* \). In addition, \( F^{*T} \) is complex conjugated vector-line, and upper subscription \( T \) indicates matrix transposition. Since every DOE at the central frequency has a unique set of diffractive orders, the correlation between beams will be only the terms that appear due to wavelength shift, that is \( F_{00} \cos(\varphi) \). It is clear that the coherence matrix \( M \) with order of \( N_0 = N \cdot N \) can be written as follows:
where $b = \cos(\varphi)$, $M = \{F_i^*F_i\}$, $n = iN + j$, $m = kN + l$, where $n$, $m$, $i$, $j$, $k$, $l$ are positive integer numbers in the range between 0 and $N-1$. The eigenvalues of $M$ matrix for decomposing the illumination field on the set of decorrelated fields of different intensity have to be obtained to derive a formula for $SC$. Hence, we have to obtain the solution of the following equation:

$$\det \begin{pmatrix} 1 - \varepsilon & b & b & \cdots & b \\ b & 1 - \varepsilon & b^2 & \cdots & b^2 \\ b & b^2 & 1 - \varepsilon & \cdots & b^2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ b & b^2 & b^2 & 1 - \varepsilon & \cdots \\ b^2 & b^2 & b^2 & \cdots & 1 - \varepsilon \end{pmatrix} = 0.$$  \hspace{1cm} (38)

The solution of Eq. (38) can be expressed as (see Appendix for detail derivations):

$$\varepsilon_1 = \frac{1}{2} \left[ 2 + (N_0 - 2)b^2 \pm \sqrt{(N_0 - 2)^2b^4 + 4(N_0 - 1)b^2} \right],$$

$$\varepsilon_2 = \varepsilon_3 = \cdots = \varepsilon_{N_0} = 1 - b^2.$$  \hspace{1cm} (39)

where $\varepsilon$ stands for the eigenvalues. After substitution of Eq. (39) into Eq. (3), $SC$ can be written as follows:

$$SC = \sqrt{\frac{1}{N_0} + \frac{2(N_0 - 1)\cos^2(\varphi)}{N_0^2} + \frac{(N_0 - 2)^2\cos^4(\varphi)}{N_0^2}}.$$  \hspace{1cm} (40)

A slightly different coherence matrix $M$ can be obtained, if we exclude the parallel plate DOE structure (that is DOE with only zero diffraction order beam) from the DOE sequence used for speckle suppression:

$$M = \begin{pmatrix} 1 & b^2 & b^2 & \cdots & b^2 \\ b^2 & 1 & b^2 & \cdots & b^2 \\ b^2 & b^2 & 1 & \cdots & b^2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ b^2 & b^2 & b^2 & \cdots & 1 \end{pmatrix}.$$  \hspace{1cm} (41)

where $M = \{F_i^*F_i\}$, $n = iN + j$, $m = kN + l$, where $n$, $m$, $i$, $j$, $k$, $l$ are positive integer numbers. This coherence matrix has the following eigenvalues (see Appendix):

$$\varepsilon_1 = 1 + (N_0 - 1)b^2,$$

$$\varepsilon_2 = \varepsilon_3 = \cdots = \varepsilon_{N_0} = 1 - b^2.$$  \hspace{1cm} (42)

After substitution of Eq. (42) into Eq. (3), we obtain the formula for $SC$:

$$SC = \sqrt{\frac{1}{N_0} + \frac{2(N_0 - 1)\cos^2(\varphi)}{N_0^2} + \frac{(N_0 - 2)^2\cos^4(\varphi)}{N_0^2}}.$$

$$SC = \sqrt{\frac{b^4(N_0 - 1)}{N_0 + 1}}.$$  \hspace{1cm} (43)
4.2.2 TSDOE based on Hadamard matrix

We assume that a TSDOE consists of two 1D DOEs, which together at the central wavelength, give phase distribution of the Sylvester-Hadamard based 2D DOE ($N_0 = 2^{2m}$, $N = 2^m$). The first 1D DOE has land and groove stretched along x axis and the second one has along y axis. Therefore, diffraction orders of the collimated laser diffracted by the first DOE propagate in the $oxz$ plane, and those diffracted by the second one in the $oyz$ plane. The field amplitude at the rear side of the first 1D DOE can be written as follows:

$$E_{nx} = \exp(i\psi_n)\sin(\varphi)S_{nx}(x, z)+\cos(\varphi). \quad (44)$$

where $S_{nx}(x, z)$ is the field distribution of given 1D DOE structure at the central wavelength (having unit intensity), $\psi_n$ is phase shift arising from wavelength shift from the central wavelength, and cosine term gives the amplitude of zero diffracted order. It is supposed that every DOE used in speckle suppression has a special structure, which at the central wavelength has unique set of diffraction orders so that every DOE produces uncorrelated beam that creates uncorrelated speckle pattern. To obtain a solution, we suppose that all DOEs have no zero order components at the central wavelength. It is certainly possible to derive formula for a more general case, which is, however, not included in the current paper due to the length limit. In the approximation of small divergence angle of beam diffracted on the first DOE, the field at the back plane of second DOE can be written as follows:

$$E_{ny} = E_{nx}E_{ny}^* = (\exp(i\psi_n)\sin(\varphi)S_{nx}(x, z)+\cos(\varphi))(\exp(-i\psi_n)\sin(\varphi)S_{ny}^*(y, z)+\cos(\varphi)) = \exp(i(\psi_n - \psi_n))\sin^2(\varphi)S_{nx}(x, z)S_{ny}^*(y, z) + \sin(\varphi)\cos(\varphi)(S_{nx}(x, z)\exp(i\psi_n) + S_{ny}^*(y, z)\exp(-i\psi_n)) + \cos^2(\varphi). \quad (45)$$

Since every DOE at the central wavelength has a unique set of diffractive orders, correlation between beam fields only depends on the terms that appear due to wavelength shift, that is $\cos^2(\varphi)$ and $\sin(\varphi)\cos(\varphi)(S_{nx}(x, z)\exp(i\psi_n) + S_{ny}^*(y, z)\exp(-i\psi_n))$.

Therefore, the correlation between fields in eye using different DOE structures can be written as follows:

$$\langle E_{nx}E_{ny}^* \rangle = \begin{cases} 1; & n=i, m=j \\ \cos^2(\varphi)+\sin^2(\varphi)\cos^2(\varphi)(\delta_n + \delta_m); & n \neq i \text{ or } m \neq j \end{cases} \quad (46)$$

The coherent matrix $M$ with the order of $N_0 = N*N = 2^{2m}$ for this case can be written as follows:

$$M = \begin{pmatrix} A & B & B & B & \cdots & B \ B & A & B & B & \cdots & B \ B & B & A & B & \cdots & B \ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \ B & B & B & A & \cdots & A \ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \ B & B & B & A & \cdots & A \ \end{pmatrix} \quad (47)$$

where the matrices $A$ and $B$ are submatrices with order of $N = 2^m$, $M_{nm} = \langle S_{ij}^*S_{kl}^* \rangle$, $n = i\cdot N + j$, $m = k\cdot N + l$, where $n, m, i, j, k, l$ are positive integer numbers in the range from 0 to $N-1$. 


and where \( s = \sin(\phi) \). The matrix \( M \) has the eigenvalues as follows (see Appendix):

\[
\begin{align*}
\varepsilon_1 &= 1 - (b^4 + 2b^2b^2) \quad \text{for } (N-1)(N-1) \text{ times degenerate,} \\
\varepsilon_2 &= 1 - b^4 + (N-2)b^2 \\
\varepsilon_3 &= 1 + 2(N-1)s^2b^2 + (N-1)(N+1)b^4.
\end{align*}
\]

By substituting the obtained eigenvalues into Eq. (3), we obtain \( SC \) formula as:

\[
SC = \sqrt{1/N^2 + 2s^2b^2(N-1)^2 / N^2 + 2s^2b^4(N-1)(5N-6) / N^4 + 4s^2b^6(N-1)(N-1)(N+1) / N^2 + b^8(N-1)(N-1) / N^2}. \tag{51}
\]

5. Results of numerical simulation and discussion

5.1 Dispersion of speckle suppression efficiency

We derive the generalized formulae of speckle suppression efficiency for the speckle suppression method using different active one-sided 2D DOE (OSDOE) and two-sided 1D DOE (TSDOE) structures. The formula of \( SC \) is obtained for two different kinds of active DOE: (1) linear or step-wise DOE shift used for activating DOE structure; (2) switching between different DOE structures by using active optical medium. In several cases, the assumption that DOE has zero intensity for zero diffractive order at the central wavelength was used to derive the analytical formula for \( SC \). However, it was found that for DOE based on a relatively small binary pseudorandom code length \( N \leq 5 \), the zeroth order intensity at the central wavelength should be taken into account for the accuracy of \( SC \) calculation. We do not use specific properties of OSDOE based on binary pseudorandom sequences or Hadamard matrix during the derivation for the \( SC \) formula except those required for high efficiency of speckle suppression. For the method using TSDOE structures, we imposed condition on the number of DOE structure \( N_0 = 2^m \) used for speckle suppression. During the following numerical simulation, we add a factor of \( 1/\sqrt{2} \) to the obtained formulae of \( SC \) to take into account the speckle suppression effect conditioned by the depolarization of laser fields scattered from the rough surface of screen [25].

Figure 5 shows the dispersion of \( SC \) for the method using linear shift of 2D and two-sided 1D DOE based on binary pseudorandom sequences with different code length (different number of decorrelated beams). Figure 5 presents the results calculated by the formulae obtained above and by the mathematical model [15] that uses autocorrelation function of moving along the screen laser beam, which has already been experimentally validated [5]. By the comparison of dispersion curves presented in Fig. 5, one can see that the results of \( SC \) obtained with different methods match well.

All numerical results presented in Fig. 5 show that the method using TSDOE allows a larger waveband range of speckle suppression than the one with OSDOE. However, for TSDOE with a short code length \( N \leq 6 \) or \( N_0 \leq 36 \), the difference in waveband is not significant for practical applications since both DOE structures allow effectively suppressed speckle in the entire visible range. For DOE with larger speckle suppression efficiency (for \( N \)
> 6), the waveband range of the method using OSDOE decreases and becomes smaller than the visible range. So $SC$ at the boundary of the visible range is higher than that at the central wavelength by 20%, 100% and >300% for $N = 7, 15$ and 31, respectively, even without taking into account the dispersion of the DOE medium. Since we only consider the number of diffraction orders for analysis of $SC$ dispersion (only for small code length we use specific properties of OSDOE based on pseudorandom sequences), it should be expected that all highly efficient methods for speckle suppression using one-sided DOE shift have a speckle suppression range smaller than the visible range of human eye.

From Fig. 5, it is also clear that the method using TSDOE structure has a wider waveband of efficient speckle suppression than the visible range, independent of its efficiency of speckle suppression. Therefore, only one DOE can be used for efficient speckle suppression in the entire visible range.

Figure 6 shows the dispersion of $SC$ for the method using DOE switch to decorrelate light intensities such as the method based on Sylvester-Hadamard matrix for 2D and two-sided 1D DOE structures. For 2D DOE (OSDOE) structure, Fig. 6 presents the results of $SC$ simulation for the case when the DOE set includes parallel plate (DOE with only zero order beam) and without it. From a comparison of $SC$ results for the two cases, we can see that they have almost same dispersion. Therefore, neglecting zero order for DOE structure, which is used in mathematical model of the method based on TSDOE, should not lead to significantly different results. From comparison of the results presented in Fig. 5 and Fig. 6, it is clear that the methods using DOE shift and DOE switch have practically the same dispersion. An
OSDOE for the case of \( N_0 > 36 \) (or \( N > 6 \)) has a smaller speckle suppression range than the visible range. The method using a TSDOE structure has a wider speckle suppression range than the visible range. Therefore, only one DOE can be used to suppress SC in entire visible range. On the other hand, the method based on 2D DOE structure with a large speckle suppression effect has a smaller speckle suppression range than the visible range sensed by human eyes.

![Image](image_url)

**Fig. 6.** Dispersion of speckle contrast for the method using DOE based on Sylvester-Hadamard matrix with different matrix order.

### 5.2 Effective number of diffraction orders for different methods of DOE activation

Accurate calculation of speckle suppression efficiency requires the knowledge of number of the decorrelated laser beams that illuminate the screen. The number of decorrelated beams depends on the method of DOE activation and the numerical aperture of the objective lens. The different methods of changing DOE phase relief such as linear DOE shift, step-wise DOE shift with one elementary cell, and switching of DOE structures in spatial time sequence produce different number of effective diffraction orders. The different methods, therefore, should have different efficiency of speckle suppression as well as different dependence of speckle suppression efficiency on the numerical aperture. Understanding the dependence of speckle suppression efficiency on different DOE activation and numerical aperture is very important for the optimization of speckle suppression mechanism.

The width of autocorrelation peaks of the binary pseudorandom sequence is practically same as the width of the autocorrelation peak of rectangular function, in which the rectangular width equals the width of DOE elementary cell \( T \) (see Fig. 2). Since the autocorrelation function is determined by the intensity of spatial frequencies, the intensity of diffraction orders of DOE based on binary pseudorandom sequences should be practically the same as the intensity of spatial spectrum of the rectangular function:

\[
I_n = \sin c^2 \left( \frac{n \pi}{N} \right).
\]  

where \( \sin c(x) = \sin(x)/x \).
Figure 7 shows the simulation results of the intensity of diffraction orders of DOE based on $M$-sequence with code length $N = 15$, from which it is clear that the intensity of diffraction orders are correctly approximated by $\text{sinc}^2(\frac{n_{\text{dif}} \pi}{N})$, where $n_{\text{dif}}$ is the diffraction order. Figure 8 shows a measurement result, in which the photograph of diffraction orders and cross-section of intensity distribution for OSDOE based on Barker code with code length $N = 13$ are presented. Although the intensity in Fig. 8 is largely modulated by speckles, it is clear that the intensity of diffraction orders is approximately fitted with $\text{sinc}^2$ function. Thus, we can use the approximation to analyze the efficiency of SC with the method based on pseudorandom binary DOE. To analyze the influence of numerical aperture on speckle suppression efficiency, it should be assumed that the projector has a finite aperture of objective lens and therefore it truncates some higher diffraction orders.

Equations (7) and (12) were used to calculate the intensity of the decorrelated laser beams for linear and step-wise shifts, respectively. Only the diffraction orders that passed through the objective diaphragm were taken into account in Eqs. (7) and (12). Table 1 shows the effective numbers ($N_{\text{ef}}$) of the decorrelated laser beams for linear 2D DOE movement with a small inclination angle and for step-wise DOE shift, which are calculated by Eq. (5) and Eq. (12) for different input numerical aperture ($NA$) of projector. From the data presented in Table 1, it is clear that in the case of large input aperture ($NA \gg \frac{\lambda}{T}$) of the objective lens, the linear shift gives SC of $1/(1.5*N)$, which is significantly better than the SC ($1/N$) obtained by a step-wise shift. The result coincides with the one obtained earlier by using autocorrelation function for SC calculation [18-19]. SC of $1/N$ for DOE with step-wise shift is predictable since during intensity integration time, DOE takes only $N^2$ fixed position that can give at most $N^2$
decorrelated laser beams \((SC \geq 1/\sqrt{N2})\). With the decrease of numerical aperture, the number of decorrelated laser beams \(N_{ef}\) for linear DOE shift slowly decrease. However, it does not change for step-wise shift until \(NA < 0.5\lambda/T\). For a small numerical aperture of \(NA < 0.5\lambda/T\), both methods give almost the same value of \(SC\) (for the same \(N_{ef}\)). This can be explained as that for a small numerical aperture \((NA \leq 0.5\lambda/T)\), the field of any decorrelated laser beam for step-wise DOE shift degenerates to the field of one diffraction order and therefore both methods have the same number of decorrelated laser beams.

According to the above analysis, it is sufficient to use an objective lens with a relatively low numerical aperture \((NA \approx 0.5\lambda/T)\) to obtain practically maximum speckle suppression effect for the method using step-wise shift of DOEs. A small numerical aperture, however, results in smaller optical efficiency (the intensity decreases to around 20\%). For the method using linear DOE shift, significantly lower \(SC\) is obtained at the price of using a large numerical aperture and faster shift speed of DOE \((L/NT_0>>1)\).

The analysis of Sylvester-Hadamard based DOE structure shows that all generated DOE structures have a diffraction divergent angle smaller or equal to \(0.5\lambda/T\) and therefore should have similar dependence of speckle suppression efficiency on numerical aperture as the Barker-code based DOE with step-wise shift.

### Table 1. Dependence of effective diffraction orders on input numerical aperture of a laser projector

<table>
<thead>
<tr>
<th>(NA)</th>
<th>(0.1\ \lambda/T)</th>
<th>(0.25\ \lambda/T)</th>
<th>(0.4\ \lambda/T)</th>
<th>(0.5\ \lambda/T)</th>
<th>(0.8\ \lambda/T)</th>
<th>(\lambda/T)</th>
<th>(2\ \lambda/T)</th>
<th>(3\ \lambda/T)</th>
<th>(\infty)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N_{ef}) (continuous shift)</td>
<td>0.2N</td>
<td>0.5N</td>
<td>0.78N</td>
<td>0.95N</td>
<td>1.21N</td>
<td>1.23N</td>
<td>1.35N</td>
<td>1.4N</td>
<td>1.49N</td>
</tr>
<tr>
<td>(N_{ef}) (step-wise shift)</td>
<td>0.2N</td>
<td>0.5N</td>
<td>0.78N</td>
<td>0.95N</td>
<td>0.95N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
</tbody>
</table>

### 6. Conclusion

A new approach for theoretical description of speckle suppression efficiency based on active DOE is proposed in the current paper. The approach is based on spectral analysis of diffracted beams and coherent matrix. Analytical formulae are obtained for calculating the dispersion of speckle suppression efficiency for methods using different DOE structures and with different activation methods of DOE.

The dispersion of speckle suppression efficiency for different binary DOE structures was investigated. It was shown that TSDOE has always broader speckle suppression range than OSDOE. It was found that the methods based on OSDOE structure (with sufficiently large code length \(N>5\) and with large speckle suppression coefficient \(k>5\)) has a speckle suppression range smaller than the visible range of human eyes. Therefore, only one OSDOE structure cannot be used for efficient speckle suppression of red, green and violet lasers. Analysis of methods based on TSDOE has shown that they have speckle suppression range wider than the visible range for any speckle suppression efficiency. Only one TSDOE structure, therefore, can be used for efficient speckle suppression in the entire visible range.

The speckle suppression efficiency for the methods using linear DOE shift and step-wise DOE shift were respectively analyzed. Switching of DOE structures can be achieved by mechanical step-wise shift or electrically controlling the optical active medium. It was shown that the method using linear 2D DOE movement can provide significantly smaller \(SC\) than the method with step-wise DOE shift (lower by a factor 1/1.5). However, it comes at the price of fast DOE speed \((L/NT_0>>1)\) and a large input numerical aperture \((NA>>\lambda/T)\) of the projector objective lens. Both methods have approximately the same suppression efficiency for a small aperture of \(NA<\lambda/2T\). The speckle suppression efficiency of the method using step-wise DOE shift remains almost constant with the decrease of input numerical aperture of the projector objective lens till the numerical aperture decreases below \(\lambda/2T\).
Appendix

In order to obtain the solution of Eq. (38) in the main text and calculate the decorrelated intensities for 2D DOE, we start from Eq. (53):

\[
\text{det}(M) = \text{det} \begin{pmatrix} u & b & b & \cdots & b \\ b & u & b^2 & \cdots & b^2 \\ b & b^2 & u & \cdots & b^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b & b^2 & b^2 & \cdots & u \end{pmatrix} = 0. \quad (53)
\]

where \( u = 1 - \epsilon \), \( \epsilon \) is required intensity, and matrix \( M \) has order of \( N_0 = N \cdot N \). By multiplying the first row by \( b \) and extracting it from all other matrix rows, the problem is simplified to the calculation of determinant of a simple matrix:

\[
\text{det} \begin{pmatrix} u & b & b & \cdots & b \\ b - ub & u - b^2 & 0 & 0 & \cdots & 0 \\ b - ub & 0 & u - b^2 & 0 & \cdots & 0 \\ b - ub & 0 & 0 & u - b^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b - ub & 0 & 0 & 0 & \cdots & u - b^2 \end{pmatrix} = (u - b^2)^{N-1}\left[u(u - b^2) - (N_0 - 1)b(b - ub)\right] = 0.
\]

The eigenvalues of matrix \( M \) can be easily obtained from the solution of Eq. (54):

\[
\epsilon_{1,2} = \left[2 + (N_0 - 2)b^2 \pm \sqrt{(N_0 - 2)^2 b^4 + 4(N_0 - 1)b^2} \right]/2, \quad (55)
\]

\[
\epsilon_3 = \epsilon_4 = \cdots = \epsilon_{N+1} = 1 - b^2.
\]

Similarly, the eigenvalue problem for the matrix of Eq. (41) in the main text can be simplified to the following equation:

\[
\text{det}(M) = \text{det} \begin{pmatrix} u & b^2 & b^2 & b^2 & b^2 & \cdots & b^2 \\ b^2 & u & b^2 & b^2 & b^2 & \cdots & b^2 \\ b^2 & b^2 & u & b^2 & b^2 & \cdots & b^2 \\ b^2 & b^2 & b^2 & u & b^2 & \cdots & b^2 \\ b^2 & b^2 & b^2 & b^2 & u & \cdots & b^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b^2 & b^2 & b^2 & b^2 & b^2 & \cdots & u \end{pmatrix}, \quad (56)
\]

By using the same approach as for the solution of Eq. (53), Eq. (57) can be easily simplified to:

\[
\text{det}(M) = u\left(u - b^2\right)^{N_0-1} + (N_0 - 1)b^2\left(u - b^2\right)^{N_0-1} = 0. \quad (57)
\]

The solution of Eq. (57) gives the eigenvalues as follows:

\[
\epsilon_1 = 1 + (N_0 - 1)b^2, \\
\epsilon_2 = \epsilon_3 = \cdots = \epsilon_{N_0} = 1 - b^2. \quad (58)
\]

In the case of the method using the two-sided 1D structure, the calculation of intensities of the decorrelated laser beams (eigenvalues of coherence matrix in Eq. (47)) is reduced to a solution of the following equation:
where $A$ and $B$ are square matrices with the order of $N = \sqrt{N_0} = 2^m$, and one column and one row of matrix $M$ has $N$ submatrices.

$$A = \begin{bmatrix} 1 - \epsilon & b^4 + s^2b^2 & b^4 + s^2b^2 & \cdots & b^4 + s^2b^2 \\ b^4 + s^2b^2 & 1 - \epsilon & b^4 + s^2b^2 & \cdots & b^4 + s^2b^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b^4 + s^2b^2 & b^4 + s^2b^2 & b^4 + s^2b^2 & \cdots & 1 - \epsilon \end{bmatrix}.$$  

$$B = \begin{bmatrix} b^4 + s^2b^2 & b^4 & b^4 & \cdots & b^4 \\ b^4 & b^4 + s^2b^2 & b^4 & \cdots & b^4 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b^4 & b^4 & b^4 & \cdots & b^4 \end{bmatrix}.$$  

It is well known [27-28] that the determinant of block matrix $M$ having structure of $M = \begin{bmatrix} D & C \\ C & D \end{bmatrix}$, where $D$ and $C$ are square matrices with same order, can be calculated as follows:

$$\det[M] = \det[D] \cdot \det[D+C] \cdot \det[D-C].$$  

It is easy to see that matrix $M$ in Eq. (59) has the required structure with the following matrices $D$ and $C$:

$$D = \begin{bmatrix} A & B & \cdots & B \\ B & A & \cdots & B \\ \vdots & \vdots & \ddots & \vdots \\ B & B & \cdots & A \end{bmatrix},$$

$$C = \begin{bmatrix} B & B & \cdots & B \\ B & B & \cdots & B \\ \vdots & \vdots & \ddots & \vdots \\ B & B & \cdots & B \end{bmatrix}.$$
\[
D - C = \begin{bmatrix}
A - B & 0 & 0 & \cdots & 0 \\
0 & A - B & 0 & \cdots & 0 \\
0 & 0 & A - B & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & A - B
\end{bmatrix}, 
\]

(64)

\[
D + C = \begin{bmatrix}
A & B_1 & B_1 & \cdots & B_1 \\
B_1 & A & B_1 & \cdots & B_1 \\
B_1 & B_1 & A & \cdots & B_1 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
B_1 & B_1 & B_1 & \cdots & A
\end{bmatrix},
\]

where \( A_i = A + B; B_1 = B + B \):

\[
A - B = \begin{bmatrix}
1 - b^4 - s^4 b^2 - e & s^4 b^2 & s^4 b^2 & \cdots & s^4 b^2 \\
1 - b^4 - s^4 b^2 - e & s^4 b^2 & s^4 b^2 & \cdots & s^4 b^2 \\
s^4 b^2 & 1 - b^4 - s^4 b^2 - e & s^4 b^2 & \cdots & s^4 b^2 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
s^4 b^2 & s^4 b^2 & s^4 b^2 & \cdots & 1 - b^4 - s^4 b^2 - e
\end{bmatrix}, 
\]

(65)

\[
A = A + B = \begin{bmatrix}
1 + b^4 + s^4 b^2 - e & 2b^4 + s^2 b^2 & 2b^4 + s^2 b^2 & \cdots & 2b^4 + s^2 b^2 \\
2b^4 + s^2 b^2 & 1 + b^4 + s^4 b^2 - e & 2b^4 + s^2 b^2 & \cdots & 2b^4 + s^2 b^2 \\
2b^4 + s^2 b^2 & 2b^4 + s^2 b^2 & 1 + b^4 + s^4 b^2 - e & \cdots & 2b^4 + s^2 b^2 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
2b^4 + s^2 b^2 & 2b^4 + s^2 b^2 & 2b^4 + s^2 b^2 & \cdots & 1 + b^4 + s^2 b^2 - e
\end{bmatrix},
\]

(66)

\[
B_i = 2B = \begin{bmatrix}
2(1 + b^4) & 2b^4 & 2b^4 & \cdots & 2b^4 \\
2b^4 & 2(1 + b^4) & 2b^4 & \cdots & 2b^4 \\
2b^4 & 2b^4 & 2(1 + b^4) & \cdots & 2b^4 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
2b^4 & 2b^4 & 2b^4 & \cdots & 2(1 + b^4)
\end{bmatrix},
\]

(67)

The calculation of determinant of the initial matrix is therefore simplified to:

\[
det |D - C| = det |A - B|^{\frac{N}{2}} det |D + C|,
\]

(68)

where \(A - B\) matrix has the same structure as the matrix in Eq. (53). By using the same method, the eigenvalues of \(A - B\) matrix can be obtained:

\[
\varepsilon_i = \varepsilon_2 = \cdots = \varepsilon_{N - 1} = 1 - \left( b^4 + 2s^2 b^2 \right),
\]

\[
\varepsilon_1 = 1 - b^4 + (N - 2)s^2 b^2.
\]

The same algorithm is applied to calculate the determinant of \(D + C\) matrix as well as the matrix in Eq. (59). Since \(A_i - B_i = A - B; \ D_i - C_i\) matrix has the same structure as \(D - C\) matrix with a smaller number of diagonal submatrices; therefore the calculation of its determinant can be simplified to:

\[
det |D_i - C_i| = det |A - B|^{\frac{N}{4}} det |D_i + C_i|.
\]

(70)

After applying Eq. (67) to the matrix \(M\) \(m\)-times, the calculation of determinant is simplified to:

\[
det |M| = (det |A - B|)^{\frac{N}{4}} det |A_m|.
\]

(71)

where
The matrix $A_m$ has the same structure as the matrix in Eq. (53) and therefore, by applying the same method, one can obtain the following eigenvalues set:

$$
\varepsilon_1 = \varepsilon_2 = \cdots = \varepsilon_{x_N} = 1 - b^4 + (N - 2)s^2b^2,
$$

$$
\varepsilon_N = 1 + 2(N - 1)s^2b^2 + (N - 1)(N + 1)b^4.
$$

Hence, the coherence matrix $M$ of Eq. (59) has the following eigenvalues:

$$
\varepsilon_1 = 1 - \left(b^4 + 2s^2b^2\right) \quad \text{is for} \quad (N - 1) \cdot (N - 1) \quad \text{times degenerate},
$$

$$
\varepsilon_2 = 1 - b^4 + (N - 2)s^2b^2 \quad \text{is for} \quad 2N - 1 \quad \text{times degenerate},
$$

$$
\varepsilon_3 = 1 + 2(N - 1)s^2b^2 + (N - 1)(N + 1)b^4.
$$

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